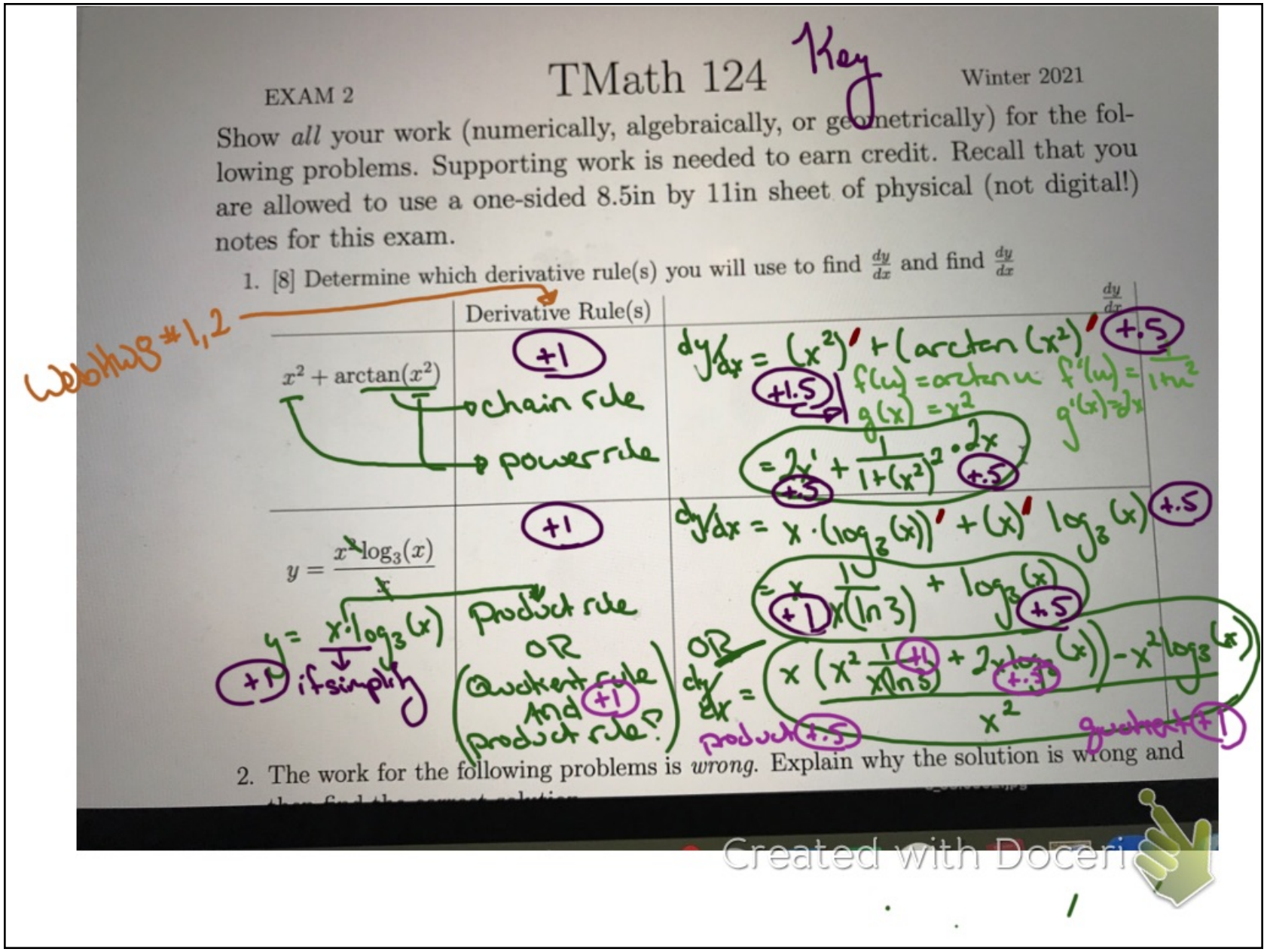


EXAM 2

TMath 124 Key

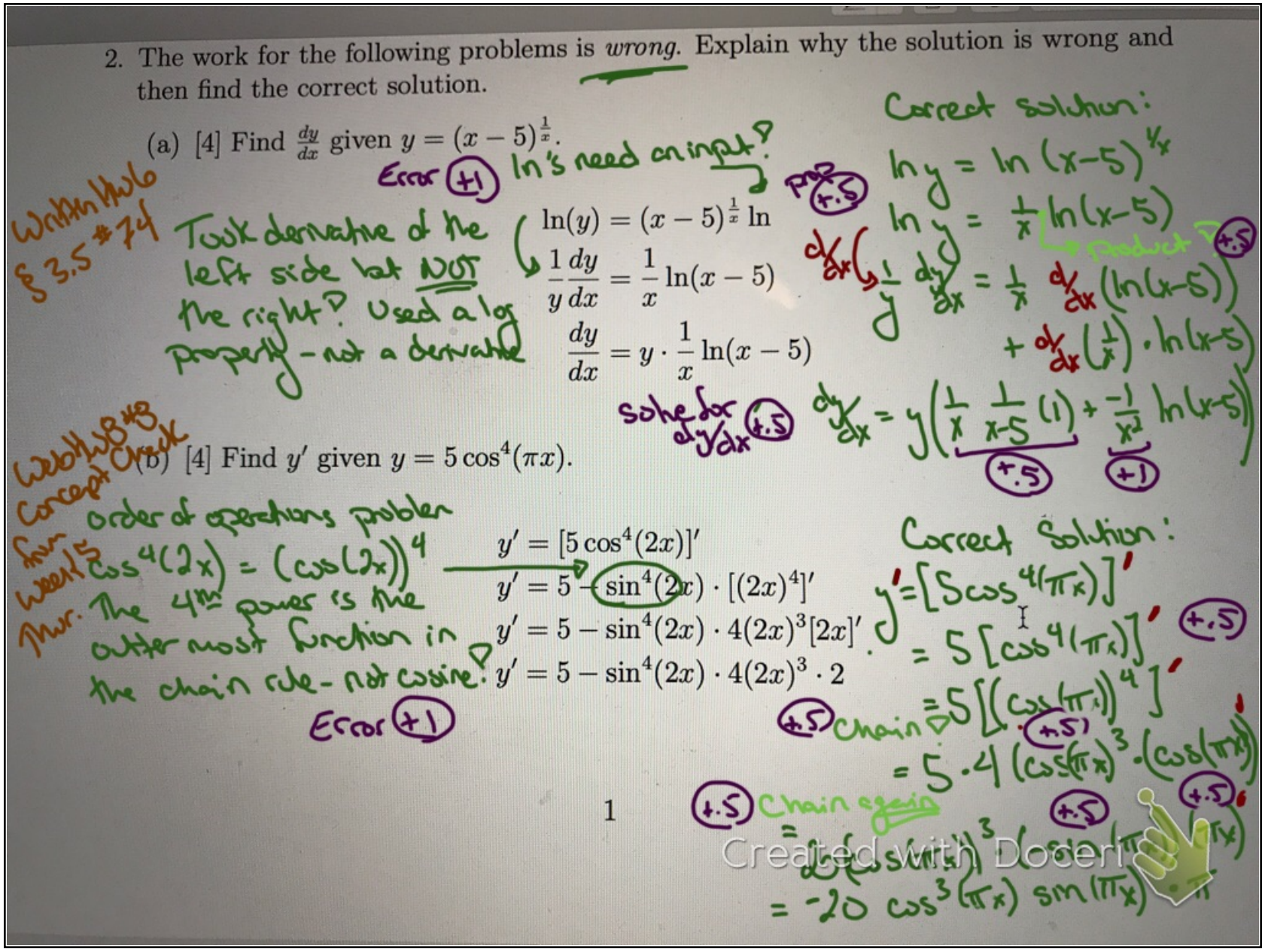
Winter 2021

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit. Recall that you are allowed to use a one-sided 8.5in by 11in sheet of physical (not digital!) notes for this exam.

1. [8] Determine which derivative rule(s) you will use to find $\frac{dy}{dx}$ and find $\frac{dy}{dx}$

	Derivative Rule(s)	$\frac{dy}{dx}$
$x^2 + \arctan(x^2)$	chain rule, power rule	$dy/dx = (x^2)' + (\arctan(x^2))'$; $f(u) = \arctan u$, $f'(u) = \frac{1}{1+u^2}$; $g(x) = x^2$, $g'(x) = 2x$; $= 2x^1 + \frac{1}{1+(x^2)^2} \cdot 2x$
$y = \frac{x^2 \log_3(x)}{x}$	Product rule OR (Quotient rule and Product rule?)	$dy/dx = x \cdot (\log_3(x))' + (x)' \log_3(x)$; $= x \cdot \frac{1}{x(\ln 3)} + \log_3(x)$; OR $dy/dx = \frac{x\left(x^2 \frac{1}{x(\ln 3)} + 2x \log_3(x)\right) - x^2 \log_3(x)}{x^2}$

2. The work for the following problems is *wrong*. Explain why the solution is wrong and



2. The work for the following problems is *wrong*. Explain why the solution is wrong and then find the correct solution.

(a) [4] Find $\frac{dy}{dx}$ given $y = (x-5)^{\frac{1}{x}}$.

Error (+1) ln's need an input?

Took derivative of the left side but NOT the right? Used a log property - not a derivative

$$\ln(y) = (x-5)^{\frac{1}{x}} \ln$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln(x-5)$$

$$\frac{dy}{dx} = y \cdot \frac{1}{x}\ln(x-5)$$

Written Hwk 3.5 #74

Correct solution:

$$\ln y = \ln(x-5)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x}\ln(x-5)$$

product

$$\frac{d}{dx}(\ln y) = \frac{1}{x}\frac{d}{dx}(\ln(x-5)) + \frac{d}{dx}\left(\frac{1}{x}\right)\cdot\ln(x-5)$$

$$\frac{dy}{dx} = y\left(\frac{1}{x}\frac{1}{x-5}(1) + \frac{-1}{x^2}\ln(x-5)\right)$$

solve for dy/dx

(b) [4] Find y' given $y = 5\cos^4(\pi x)$.

Web Hwk #3 Concept Check from Week 5 Thur. The 4th power is the outter most function in the chain rule - not cosine!

order of operations problem

$\cos^4(2x) = (\cos(2x))^4$

$$y' = [5\cos^4(2x)]'$$

$$y' = 5 - \sin^4(2x)\cdot[(2x)^4]'$$

$$y' = 5 - \sin^4(2x)\cdot 4(2x)^3[2x]'$$

$$y' = 5 - \sin^4(2x)\cdot 4(2x)^3\cdot 2$$

Error (+1)

Correct Solution:

$$y' = [5\cos^4(\pi x)]'$$

$$= 5[\cos^4(\pi x)]'$$

$$= 5[(\cos(\pi x))^4]'$$

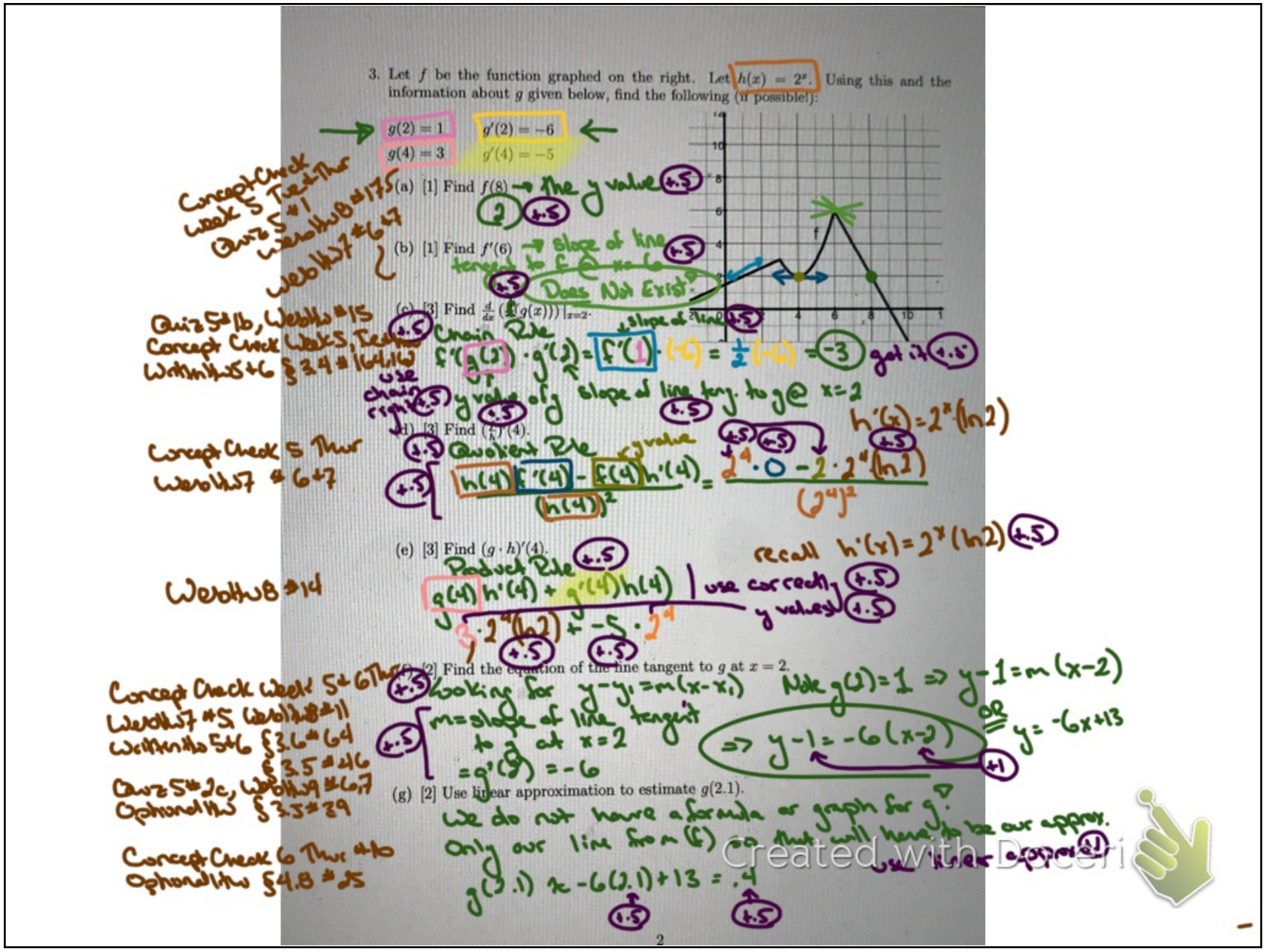
(+.5) Chain

$$= 5\cdot 4(\cos(\pi x))^3\cdot(\cos(\pi x))'$$

(+.5) Chain again

$$= -20\cos^3(\pi x)\sin(\pi x)$$

1



3. Let f be the function graphed on the right. Let $h(x) = 2^x$. Using this and the information about g given below, find the following (if possible!):

$g(2) = 1$ $\quad g'(2) = -6$

$g(4) = 3$ $\quad g'(4) = -5$

(a) [1] Find $f(8)$ → the y value +.5 ; 2 +.5

(b) [1] Find $f'(6)$ → slope of line tangent to f @ x=6 +.5 ; Does Not Exist. +.5

(c) [3] Find $\frac{d}{dx}(f(g(x)))|_{x=2}$. +.5 Chain Rule +.5 $f'(g(2)) \cdot g'(2) = f'(1) \cdot (-6) = \frac{1}{2}(-6) = -3$ +.5 get it +.5

f'(g(2)) → slope of line tang. to f @ x=1 +.5 ; g'(2) → slope of line tang. to g @ x=2 +.5

(d) [3] Find $(\frac{h}{f})'(4)$. +.5 Quotient Rule +.5 $\frac{f(4)h'(4) - h(4)f'(4)}{(f(4))^2}$ +.5 $= \frac{2^4 \cdot 0 - 2 \cdot 2^4(\ln 2)}{(2^4)^2}$ +.5 +.5 $h'(x) = 2^x(\ln 2)$

(e) [3] Find $(g \cdot h)'(4)$. +.5 Product Rule +.5 $g(4)h'(4) + g'(4)h(4)$ | use correctly +.5 ; y values +.5 recall $h'(x) = 2^x(\ln 2)$ +.5

$3 \cdot 2^4(\ln 2) + -5 \cdot 2^4$ +.5 +.5

(f) [2] Find the equation of the line tangent to g at $x = 2$. +.5 +.5 looking for $y - y_1 = m(x - x_1)$

m = slope of line tangent to g at x=2 $= g'(2) = -6$ +.5

Note $g(2) = 1 \Rightarrow y - 1 = m(x-2)$

$\Rightarrow y - 1 = -6(x-2)$ +1 OR $y = -6x + 13$

(g) [2] Use linear approximation to estimate $g(2.1)$.

We do not have a formula or graph for g.

Only our line from (f) so that will have to be our approx. +1

$g(2.1) \approx -6(2.1) + 13 = .4$ +.5 +.5

Concept Check Week 5 Tues #1 Quiz 5 #1 Webwk8 #17,5 Webwk7 #6,7

Quiz 5 #1b, Webwk8 #15 Concept Check Week 5 Tues Webwk5+6 §3.4 #16,41,46

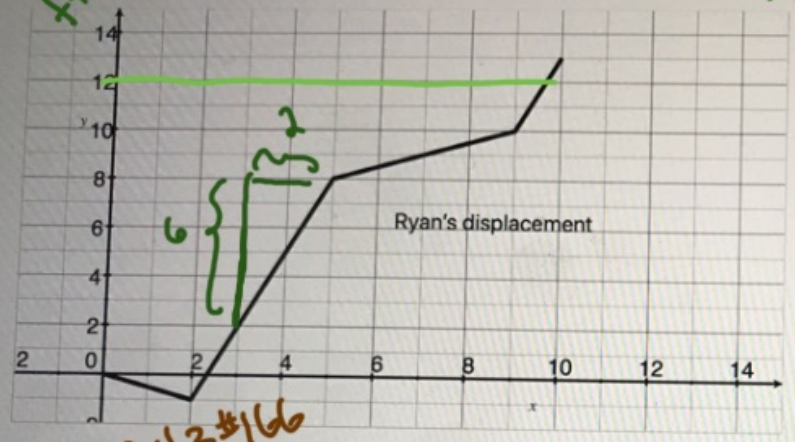
Concept Check 5 Thur Webwk7 #6,7

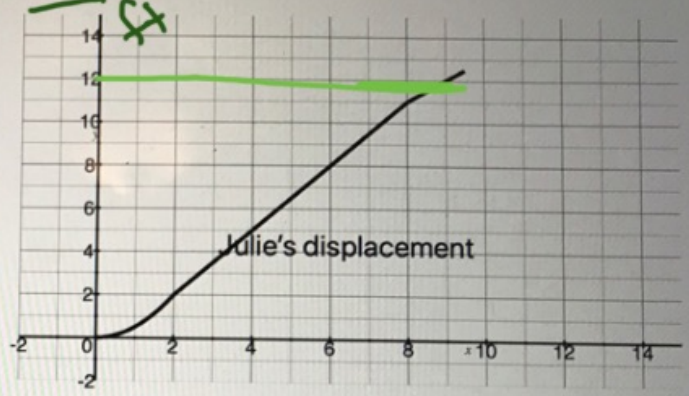
Webwk8 #14

Concept Check Week 5 & 6 Thur Webwk7 #5 Webwk8 #11 Webwk5+6 §3.6 #64 §3.5 #46 Quiz 5 #2c, Webwk9 #6,7 Optional hw §3.5 #39

Concept Check 6 Thur #10 Optional hw §4.8 #35

4. Consider the graphs recording the displacement of Dr. Ryan Card and Dr. Julie Eaton during a short race. The following graphs their respective *displacement* at time t in seconds.

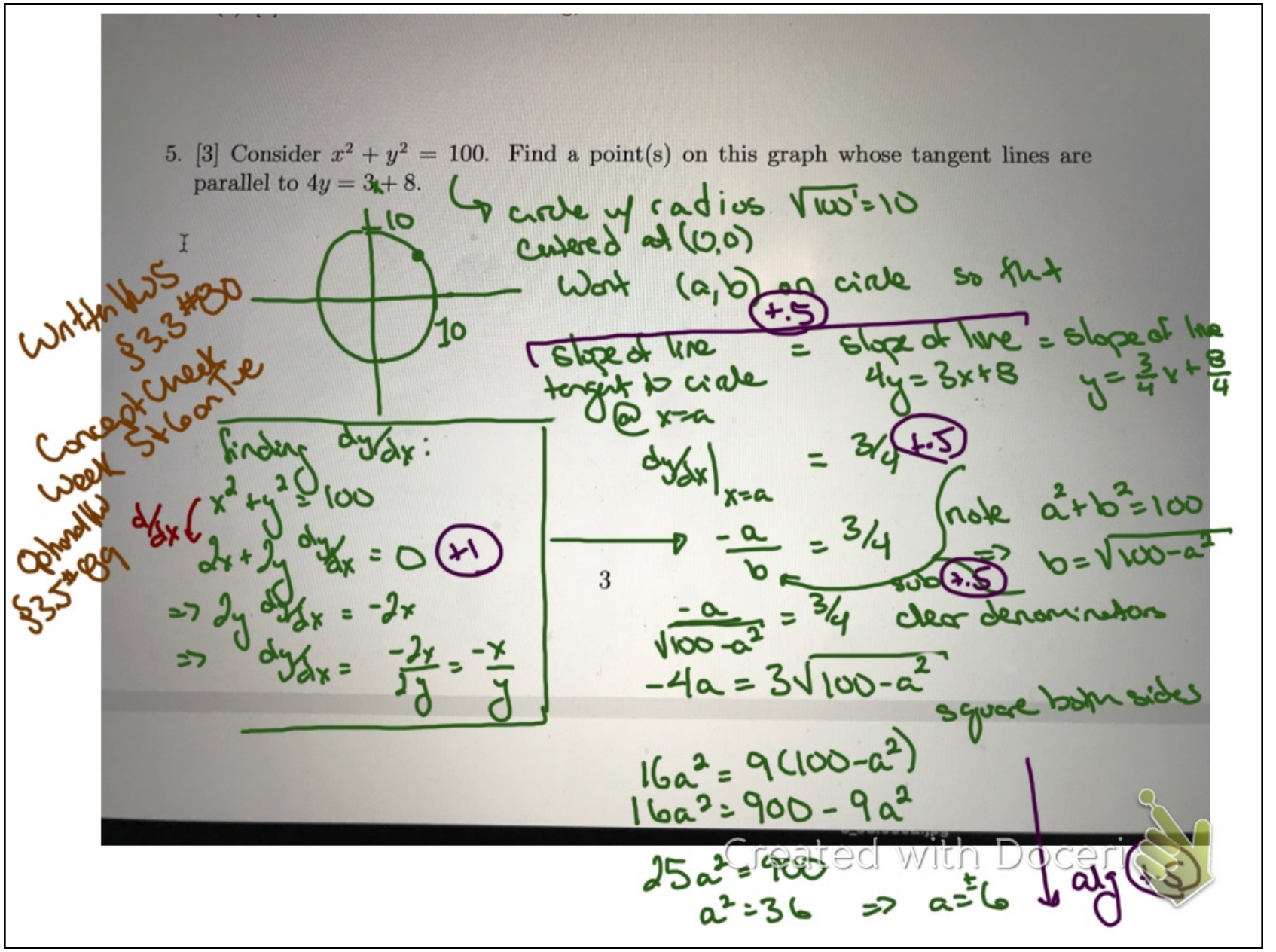




(a) [3] Who can run faster? What is his/her top speed?

(b) [2] Explain why the displacement is not increasing at a constant rate for Dr. Ryan Card.

(c) [2] If the race is 12 meters long, who wins and at what time?


5. [3] Consider $x^2 + y^2 = 100$. Find a point(s) on this graph whose tangent lines are parallel to $4y = 3x + 8$.
circle w/ radius $\sqrt{100} = 10$ centered at (0,0)
Want (a,b) on circle so that
slope of line tangent to circle @ x=a = slope of line $4y = 3x + 8$ = slope of line $y = \frac{3}{4}x + \frac{8}{4}$
+.5
finding dy/dx:
$x^2 + y^2 = 100$
$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(100)$
$2x + 2y\,dy/dx = 0$ +1
$\Rightarrow 2y\,dy/dx = -2x$
$\Rightarrow dy/dx = \frac{-2x}{2y} = \frac{-x}{y}$
$dy/dx|_{x=a} = 3/4$ +.5
$\frac{-a}{b} = 3/4$
note $a^2 + b^2 = 100$
$\Rightarrow b = \sqrt{100 - a^2}$
+2.5
$\frac{-a}{\sqrt{100-a^2}} = 3/4$ clear denominators
$-4a = 3\sqrt{100-a^2}$
square both sides
$16a^2 = 9(100 - a^2)$
$16a^2 = 900 - 9a^2$
$25a^2 = 900$
$a^2 = 36 \Rightarrow a = \pm 6$
adj +.5
3
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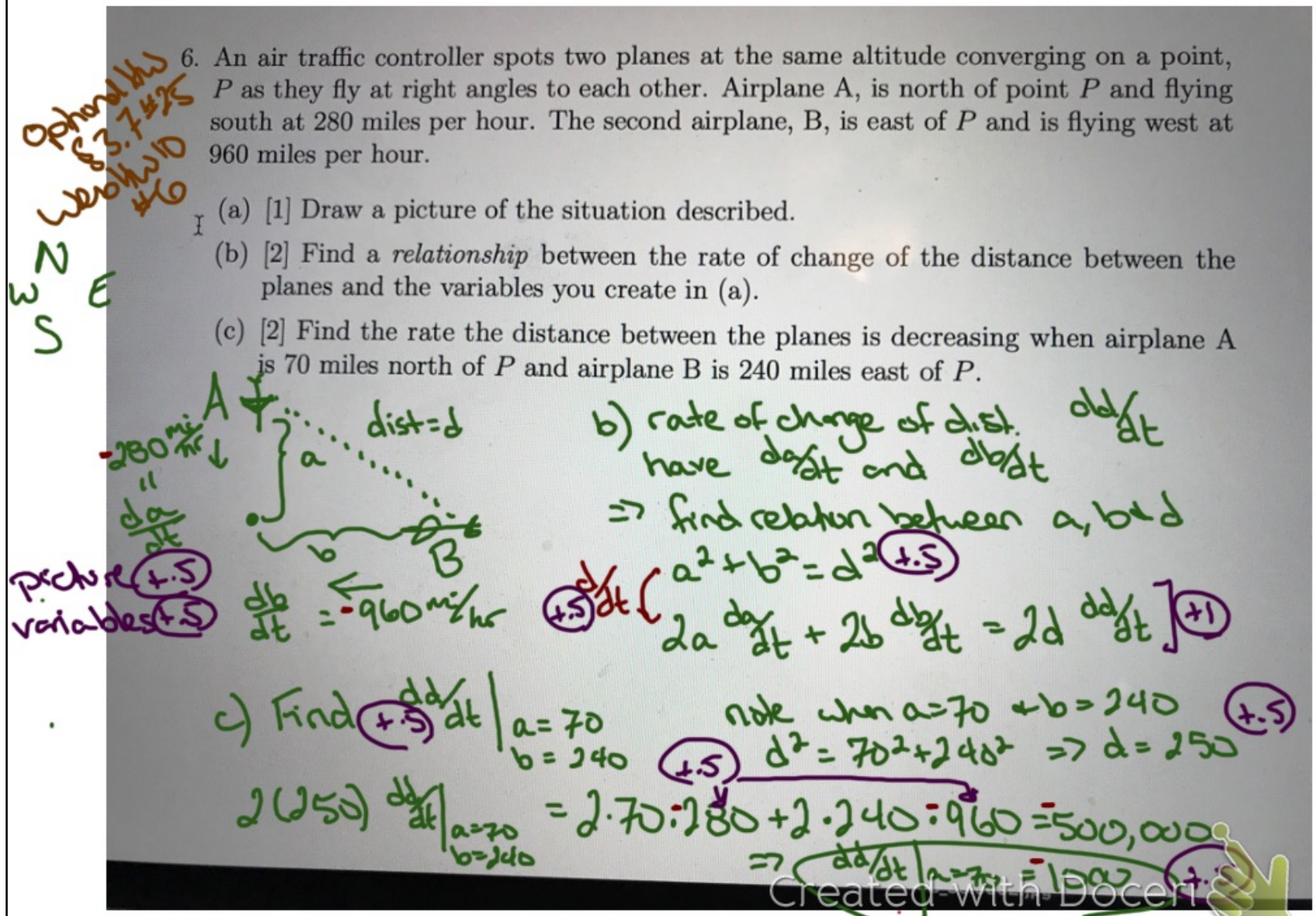
6. An air traffic controller spots two planes at the same altitude converging on a point, P as they fly at right angles to each other. Airplane A, is north of point P and flying south at 280 miles per hour. The second airplane, B, is east of P and is flying west at 960 miles per hour.

(a) [1] Draw a picture of the situation described.

(b) [2] Find a *relationship* between the rate of change of the distance between the planes and the variables you create in (a).

(c) [2] Find the rate the distance between the planes is decreasing when airplane A is 70 miles north of P and airplane B is 240 miles east of P.



Extra last problem

7. [4] Choose a problem from this exam that you've already answered,

(a) show a second way of approaching/building a solution, and

(b) explain why you did not choose this second method initially.

There are lots of right answers here?

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